XXXVI. *Account of a new Inſtrument for meaſuring ſmall Angles, called the priſmatic Micrometer. By the Rev.* Nevil Maſkelyne, *D. D. F. R. S. and Aſtronomer-Royal.*

Read Dec. 18, 1777.

PRACTICAL aſtronomy was much benefited by the invention of the wire micrometer, for meaſuring differences of right aſcenſion and declination: nor did it receive leſs advantage from Mr. SAVERY's moſt ingenious invention of the divided object-glaſs micrometer, which has been rendered more commodious by the late Mr. JOHN DOLLOND's application of it to the object-end of a reflecting teleſcope, or the preſent Mr. PETER DOLLOND's application of it to the object-end of an achromatic refracting one.

But, valuable as the object-glaſs micrometer undoubtedly is, ſome difficulties have been found in the uſe of it, owing to the alterations in the focus of the eye, which are apt to cauſe it to give different meaſures of the ſame angle at different times. For inſtance, in meaſuring the Sun's diameter, the axes of the pencils of rays, which come through the two ſegments of the object-glaſs from contrary limbs of the Sun, croſſing one another at the focus of the teleſcope under an angle equal to that of the Sun's diameter, the union of the limbs of the two images of the Sun cannot appear perfect unleſs the eye be diſpoſed to ſee objects diſtinctly which are placed at point of interſection. But if the eye be diſpoſed to ſee objects diſtinctly, which are placed nearer the object-glaſs than the interſection is, the two limbs will appear ſeparated by the interval of the axes of the pencils in that place; and if the eye be diſpoſed to ſee objects diſtinctly, which are placed farther from the object-glaſs than the interſection is, the two limbs will appear to encroach upon each other by the diſtance of the axes of the pencils, after their croſſing, taken at that place.

To explain this, let OV (plate XIX. fig. 1.) repreſent the centres of the two ſemi-circular glaſſes of the object-glaſs micrometer, ſeparated to the diſtance OV from each other, ſubtending the angle O*a*V, equal to the Sun's diameter, at the point *a*, which is the common focus of the two pencils of rays having O*a* and V*a* for their axes, namely, thoſe proceeding from contrary ſides of the Sun, and paſſing through the contrary ſemi-circles; and let *d* be the eye-glaſs. It is evident, that if *d* be properly placed to give diſtinct viſion of objects placed at the point *a*, the rays O*a*, V*a*, as well as all the other rays belonging to thoſe pencils, will be collected into one point upon the retina of the eye; and conſequently, the two oppoſite limbs of the two images of the Sun will ſeem to coincide, and the two images of the Sun to touch one another externally. But if the ſtate of the eye ſhould alter, the place of the eye-glaſs remaining the ſame, the eye will be no longer diſpoſed to ſee the image formed at the point *a* diſtinctly, but to ſee an object placed at *ef*, nearer to or farther from the object-glaſs diſtinctly; and therefore an image will be formed on the retina exactly ſimilar to the ſomewhat confuſed image formed by the rays on a plane perpendicular to their courſe at *ef*. Conſequently, as the two cones of ſolar rays, *boa*, *cva*, formed by the two ſemi-circles, are ſeparated or encroach upon one another at this point of the axis by the diſtance *ef*, the two images of the Sun will not ſeem to touch one another externally, but to ſeparate or to encroach upon one another by the interval *ef*. The error hereby introduced into the meaſure of the Sun's diameter will be the angle *erf*, ſubtended by *ef* at *r* the middle point between o and v, which is to *eaf* or o*a*v, the Sun's apparent diameter, as *ae* to *er*, or even to *ar*, on account of the ſmallneſs of *ae* with reſpect to *ar*.

Theſe conſiderations concerning the cauſe of a principal error that has been found in the object-glaſs micrometer led me to inquire, whether ſome method might not be found of producing two diſtinct repreſentations of the Sun, or any other object, which ſhould have the axes of the pencils of rays, by which they are formed, diverging from one and the ſame point, or nearly ſo: and it occurred to me, that this might be done by the refraction of a priſm placed to receive part of the rays proceeding from the object, either before or after their refraction through the object-glaſs of a teleſcope. If the priſm be placed without the object-glaſs, the rays that are refracted through it will make an angle with the rays that paſs beſide it equal to the refraction of the priſm; and this angle will not be altered by the refraction of the object-glaſs afterwards. Conſequently, two images of an object will be repreſented, and the priſm ſo applied will enable us to meaſure the apparent diameter of any object, or any other angular diſtance which is equal to the refraction of the priſm. But if the priſm be placed within the object-glaſs, that is to ſay, between the object-glaſs and eye-glaſs, the angle meaſured by the inſtrument will vary according to the diſtance of the priſm from the focus of the object-glaſs, bearing the ſame ratio to the refraction of the priſm, as the diſtance of the priſm from the focus bears to the focal length of the object-glaſs.

Let ACB (fig. 2.) repreſent the object-glaſs, and *d* the eye-glaſs of a teleſcope, and PR a priſm placed to intercept part of the rays coming from an object, ſuppoſe the Sun, before they fall on the object-glaſs. The rays EE proceeding from the Eaſtern limb of the Sun, and refracted through the object-glaſs ACB without paſſing through the priſm, will form the correſponding point of the Sun's image at *e*; and the rays WW proceeding in like manner from the Weſtern limb of the Sun will be refracted to form the correſpondent point of the Sun's image at *w*. But the rays 2E, 2E, 2W, 2W, proceeding in like manner from the Eaſtern and Weſtern limbs of the Sun, and falling on the priſm PR, and thence refracted to the object-glaſs ACB, will, after refraction through it, form the correſpondent points of the Sun's image at 2*e*, 2*w*. Let the refraction of the priſm be equal to the Sun's apparent diameter: in this caſe, at whatever diſtance the priſm be placed beyond the object-glaſs, the two images of the Sun *we*, 2*w*2*e*, will touch one another externally at the point *e*2*w*; for the rays 2W, 2W, proceeding from the Weſtern limb of the Sun being inclined to the rays EE proceeding from the Eaſtern limb in the angle of the Sun's apparent diameter, will, after ſuffering a refraction in paſſing through the priſm equal to the Sun's apparent diameter, emerge from the priſm and fall upon the object-glaſs parallel to the rays EE, and conſequently will have their focus *2w* coincident with the focus *e* of the rays EE, and therefore the two images of the Sun *we*, *2w2e*, will touch one another externally at the point *e2w*, and the inſtrument will meaſure the angle EC2W, and that only.

But if the priſm be placed within the teleſcope, the angle meaſured by the inſtrument will be to the refraction of the priſm as the diſtance of the priſm from the focus of the object-glaſs is to the focal diſtance of the object-glaſs: or if two priſms be uſed to form the two images, with their refracting angles placed contrary ways, as repreſented in fig. 3. and 4. the angle meaſured will be to the ſum of the refractions of the priſms as the diſtance of the priſms from the focus of the object-glaſs is to the focal diſtance of the object-glaſs. For let ACB (fig. 3.) repreſent the object-glaſs, and *d* the eye-glaſs of a teleſcope, and PR, RS, two priſms interpoſed between them, with their refracting angles turned contrary ways, and the common ſections of their refracting planes touching one another at R. The rays proceeding from an object, ſuppoſe the Sun, will be diſpoſed, by the refraction of the object-glaſs, to form an image of the Sun at the focus; but part of them falling on one priſm, and part on the other, will be thereby refracted contrary ways, ſo as to form two equal images *we*, *2w2e*, which, if the refractions of the priſms be of proper quantities, will touch one another externally at the point *e2w*. Let ECN be the axis of the pencil of rays EE proceeding from the Sun's Eaſtern limb; and WCO the axis of the pencil of rays WW proceeding from the Sun's Weſtern limb; and the point N the place where the image of the Sun's Eaſtern limb would be formed, and the point O where that of the Weſtern limb would be formed, were not the rays diverted from their courſe by the refractions of the priſms. But by this means part of the rays EE, which were proceeding to N, falling on the priſm PR, will be refracted to form an image of the Sun's Eaſtern limb at *e*, while others of the rays EE, which fall on the priſm RS, will be refracted to form an image of the Sun's Eaſtern limb at *2e*. In like manner, part of the rays WW, which were proceeding to form an image of the Sun's Weſtern limb at O, falling on the priſm RS, will be refracted to form an image of the Sun's Weſtern limb at *2w* coincident with *e*, the point of the image correſpondent to the Sun's Eaſtern limb; while others of the rays WW, which fall on the priſm PR, will be refracted to form the image of the Sun's Weſtern limb at *w*. The two images *we*, *2w2e*, are ſuppoſed to touch one another externally at the point *e2w*. The ray EFR, which belongs to the axis ECN, and is refracted by the priſm PR to *e*, undergoes the refraction NR*e*, which (be-cauſe ſmall angles are proportional to their ſines, and the ſine of NR*e* is equal to the ſine of its ſupplement NRC) is to NCR as NC or C*e* is to NR or R*e*. In like manner, the ray WGR, which belongs to the axis WCO, and is refracted by the priſm RS to 2*w* or *e*, undergoes the refraction OR *e*, which is to OC*e* as OC or C*e* is to RO or R*e*; therefore, by compoſition, ORN the ſum of the refractions OR*e*, NR*e*, is to OCN the ſum of the angles OC*e*, NC*e*, or the Sun's apparent diameter, as C*e* to R*e*; that is, as the focal diſtance of the object-glaſs to the diſtance of the priſms from the focus of the object-glaſs.

Or let the priſms PR, RS, be placed with their refracting angles P, S, turned from one another as in fig. 4.: the refraction of the priſm PR will transfer the image of the Sun from ON to *we*, and the refraction of the priſm RS will transfer the image ON to 2*w*2*e*, the two images 2*w*2*e*, *we*, touching one another externally at the point 2*ew*. Let ECN, WCO, be the axes of the pencils of rays proceeding from the two extreme limbs of the Sun, and N, O, the points where the images of the Sun's Eaſtern and Weſtern limbs would be formed by the object-glaſs, were it not for the refraction of the priſms; the ray EFR, which belongs to the axis ECN, and is refracted by the priſm RS to 2*e*, undergoes the refraction NR2*e*; and the ray WGR, which belongs to the axis WCO, and is refracted by the priſm PR to *w*, undergoes the refraction OR*w*. Now NC2*e*, part of the angle meaſured, is to NR2*e*, the refraction of the priſm RS, as R*w* to C*w*; and OC*w*, the other part of the angle meaſured, is to OR*w*, the refraction of the priſm PR, in the ſame ratio of R*w* to C*w*: therefore OCN, the whole angle meaſured, is to ORN, the ſum of the refractions of the two priſms, as R*w* to C*w*; that is, as the diſtance of the priſms from the focus of the object-glaſs to the focal diſtance of the object-glaſs.

When the priſms are placed in the manner repreſented in fig. 3. the point *e* of the image *we* is illuminated only by the rays which fall on the object-glaſs between A and F, and the point 2*w* only by the rays which fall on the object-glaſs between B and G. Now the angles CRF, CRG, equal to the refractions of the priſms, being conſtant, the ſpaces FC, CG, will increaſe in proportion as the diſtances RF, RG, increaſe, and the ſpaces AF, GB, diminiſh as much; and therefore, the images at the point of mutual contact *e*2*w* will be each illuminated by half the rays which fall on the object-glaſs when the priſms are placed cloſe to the object-glaſs; but will be enlightened leſs and leſs the nearer the priſms are brought to the focus of the object-glaſs.

But when the priſms are placed in the manner ſhewn in fig. 4. the images at the point of contact, as the priſms are removed from the object-glaſs towards the eye-glaſs, will be enlightened with more than half the rays that fall on the object-glaſs, and will be moſt enlightened when the priſms are brought to the focus itſelf; for the point *2e* of the image *2w2e* will be enlightened by all the rays EE that fall on the object-glaſs between B and F, and the point *w* of the image *we* will be enlightened by all the rays WW which fall on the object-glaſs between A and G. But the difference of the illuminations is not very conſiderable in achromatic teleſcopes, on account of the great aperture of the object-glaſs; as the greateſt ſpace FG is to the focal diſtance of the object-glaſs, as the ſum of the ſines of the refractions of the priſms is to the radius.

There is a third way, and perhaps the beſt, of placing the priſms, ſo as to touch one another along their ſides which are at right angles to the common ſections of their refracting planes. In this diſpoſition of the priſms, the images will be equally enlightened, namely, each with half the rays which fall on the object-glaſs, wherever the priſms be placed between the object-glaſs and eye-glaſs.

From what has been ſhewn it appears, that this inſtrument, which may be properly called the priſmatic micrometer, will meaſure any angle that does not exceed the ſum of the refractions of the priſms, excepting only very ſmall angles, which cannot be taken with it on account of the vaniſhing of the pencils of rays at the juncture of the two priſms near the focus of the object-glaſs; that it will afford a very large ſcale, namely, the whole focal length of the object-glaſs for the greateſt angle meaſured by it; and that it will never be out of adjuſtment; as the point of the ſcale where the meaſurement begins (or the point of o) anſwers to the focus of the object-glaſs, which is a fixed point for celeſtial objects, and a point very eaſily found for terreſtrial objects. All that will be neceſſary to be done, in order to find the value of the ſcale of this micrometer, will be to meaſure accurately the diſtance of the priſms from the focus when the inſtrument is ſet to meaſure the apparent diameter of any object ſubtending a known angle at the centre of the object-glaſs, which may be eaſily found by experiment, as by meaſuring a baſe and the diameter of the object obſerved placed at the end of it, in the manner practiced with other micrometers: for the angle ſubtended by this object will be to the angle ſubtended by a celeſtial object, or very remote land object, when the diſtance of the priſms from the principal focus is the ſame as it was found from the actual focus in the terreſtrial experiment, as the principal focal diſtance of the object-glaſs is to the actual focal diſtance in the ſaid experiment.

It will, I apprehend, be the beſt way in practice, inſtead of one priſm to uſe two priſms, refracting contrary ways, and ſo divide the refraction between them (as repreſented in fig. 3. and 4.). Achromatic priſms, each compoſed of two priſms of flint and crown-glaſs, placed with their refracting angles contrary ways, will undoubtedly be neceſſary for meaſuring angles with great preciſion by this inſtrument: and I can add with pleaſure, that I find by experiment made with this inſtrument, as it was executed by Mr. DOLLOND with achromatic priſms, ground with great care for this trial above a twelve-month ago, that the images after refraction through the priſms appear very diſtinct; and that obſervations of the apparent diameters of objects may be taken in the manner here propoſed with eaſe and preciſion.

Two or more ſets of priſms may be adapted to the ſame teleſcope, to be uſed each in their turn, for the more commodious meaſurement of different angles. Thus it may be very convenient to uſe one ſet of priſms for meaſuring angles not exceeding 36′, and conſequently fit for meaſuring the diameters of the Sun and Moon, and the lucid parts and diſtances of the cuſps in their eclipſes; and another ſet of priſms to meaſure angles not much exceeding one minute, and conſequently fit for meaſuring the diameters of all the other planets. This latter ſet of priſms will be the more convenient for meaſuring ſmall angles, on account of a ſmall imperfection attending the uſe of this micrometer, as before mentioned; namely, that angles cannot be meaſured with it when the priſms approach very near the focus of the object-glaſs, the pencils of rays being there loſt at the point where the priſms touch one another.

Upon the principles that have been here explained, a priſm placed within the teleſcope of an aſtronomical inſtrument, adjuſted by a plumb-line or level, to receive all the rays that paſs through the object-glaſs, may conveniently ſerve the purpoſe of a micrometer, and ſuperſede the uſe both of the vernier ſcale and the external micrometer; and the inſtrument may then be always ſet to ſome even diviſion before the obſervation. Thus the uſe of a teleſcopic level may be extended to meaſure with great accuracy the horizontal refractions, the depreſſion of the horizon of the ſea, and ſmall altitudes and depreſſions of land objects. Time and experience will doubtleſs ſuggeſt many other uſeful applications of this inſtrument.

A paper from the learned Abbé BOSCOVICH was read before this Society the ninth of laſt June, deſcribing a ſimilar contrivance as an invention of the Abbé ROCHON, in which the Abbé BOSCOVICH himſelf alſo claims ſome ſhare; I therefore deſire to acquaint this Society, that I communicated this invention to Mr. DOLLOND, and had it executed by him; and alſo ſhewed the inſtrument itſelf, ſo executed, to my eſteemed friend ALEXANDER AUBERT, eſq. fellow of this Society, a gentleman very well qualified to judge of things of this nature, above a twelvemonth before the communication of the Abbé BOSCOVICH'S paper, as will appear from their written atteſtations, drawn up at my deſire, deſcribing the particulars of the communication of this invention which I made to them ſo long ago. May I be permitted to remark, that this inſtrument having been executed by my directions, in ſeveral forms, by Mr. DOLLOND, between the months of March and Auguſt, 1776, and ſet up and tried at his houſe in the preſence of ſeveral of his workmen, could not be conſidered as an abſolute ſecret concealed from the public. However, I doubt not that the following atteſtations of Mr. AUBERT and Mr. DOLLOND will ſufficiently prove my title to this invention of the priſmatic micrometer;

meter; and I take this opportunity of exhibiting to the Society the inſtrument itſelf, mentioned in Mr. DOLLOND's letter as executed by himſelf according to my directions, and ſent to the Royal Obſervatory in the month of Auguſt 1776.

Greenwich,
December 11, 1777.

TO THE REV. DR. MASKELYNE.

St. Paul's Church-yard,
Nov. 22, 1777.

REV. SIR,

ACCORDING to your deſire I ſend the following particulars of the experiments which were made by your directions, for compleating a new kind of micrometer for meaſuring ſmall angles. About the beginning of April 1776, I received your firſt directions reſpecting this matter, which were to make two priſmatic glaſſes or wedges of ſuch angles that rays of light, which paſſed through them, ſhould be refracted about 18′ of a degree: theſe were to be placed between the object-glaſs and eye-glaſs of an achromatic teleſcope about 30 inches long. The angular edges of the two priſmatic glaſſes were to be placed in contact with each other; they were to be moved in a parallel poſition from the object-glaſs to the focus of the eye-glaſs, and to be of ſuch a ſize as to cover the aperture of the object-glaſs when brought cloſe to it. By the refraction of theſe wedges two images were formed in the teleſcope, which were at the greateſt diſtance (about 36′) when the wedges were cloſe to the object-glaſs, and approached as they were moved towards its focus, where they united; ſo that the whole focal diſtance of the object-glaſs was to be the length of the ſcale for meaſuring the angular diſtance of the two images formed in the teleſcope. When theſe wedges were applied, as above deſcribed, the two images were found to be coloured to a great degree, occaſioned by the refraction of the wedges. This defect you directed me to remove by making the priſmatic glaſſes or wedges achromatic, on the ſame principles as the achromatic object-glaſſes; and, after ſome difficulties, this was effected; the two images formed in the teleſcope appeared free from colours and diſtinct. The above experiments were made in a rough wooden tube, with an inconvenient method of moving the wedges by hand: in this ſtate it was when ſhewn to ALEXANDER AUBERT, eſq. F. R. S. towards the end of May, 1776; after which you deſired to have it done in a more compleat manner, in a braſs tube, with a means of turning the tube round to take angles in different directions, and a method of moving the wedges with a ſcrew. This was compleated about the middle of Auguſt in the ſame year, and then ſent to the Royal Obſervatory. I have the honour to be,

REVEREND SIR,

Your obedient humble ſervant,

PETER DOLLOND.

I HEREBY certify, that in the month of May, 1776, the Rev. Mr. MASKELYNE, Aſtronomer-royal, produced to me, at Mr. DOLLOND's houſe in St. Paul's church-yard, and in his preſence, as a new invention of his own, an inſtrument for meaſuring ſmall angles, conſiſting of two achromatic priſms or wedges applied between the object-glaſs and eye-glaſs of an achromatic teleſcope about 30 inches long, by moving of which wedges nearer to, or farther from, the object-glaſs, the two images of an object produced by them appeared to approach to, or recede from, each other, ſo that the focal length of the object-glaſs became a ſcale for meaſuring the angular diſtance of the two images.

London,
Nov. 27, 1777.

ALEX^R. AUBERT.

* * *

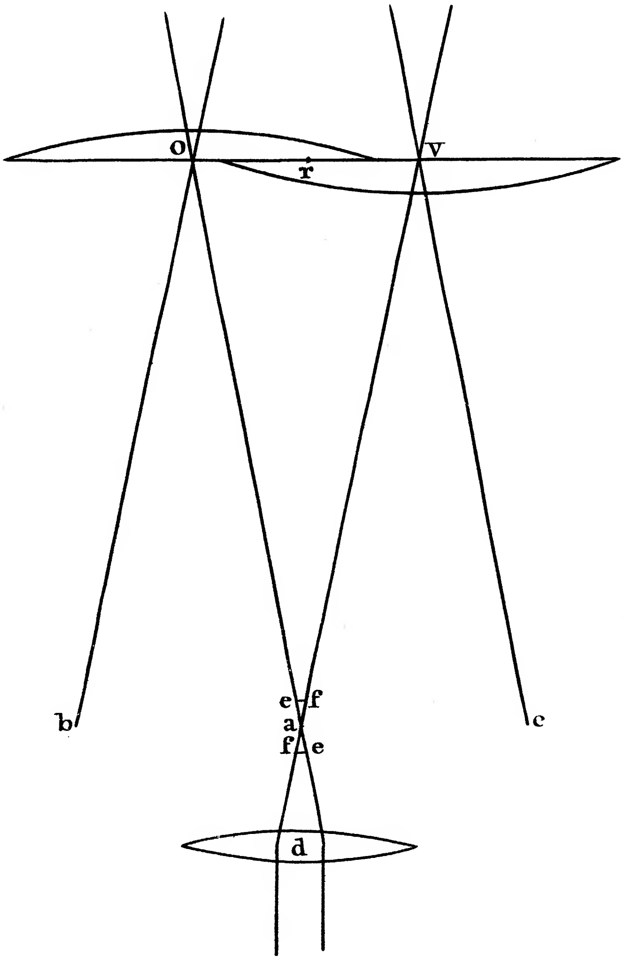


Fig: I.

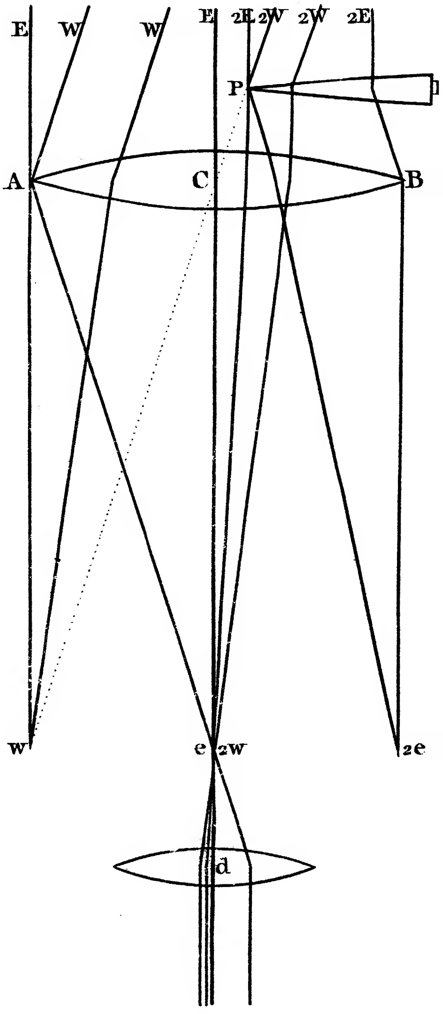
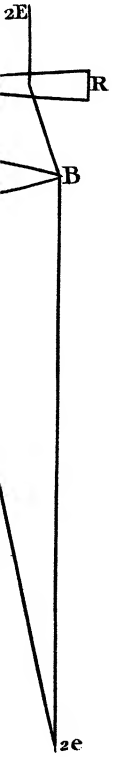


Fig: II.



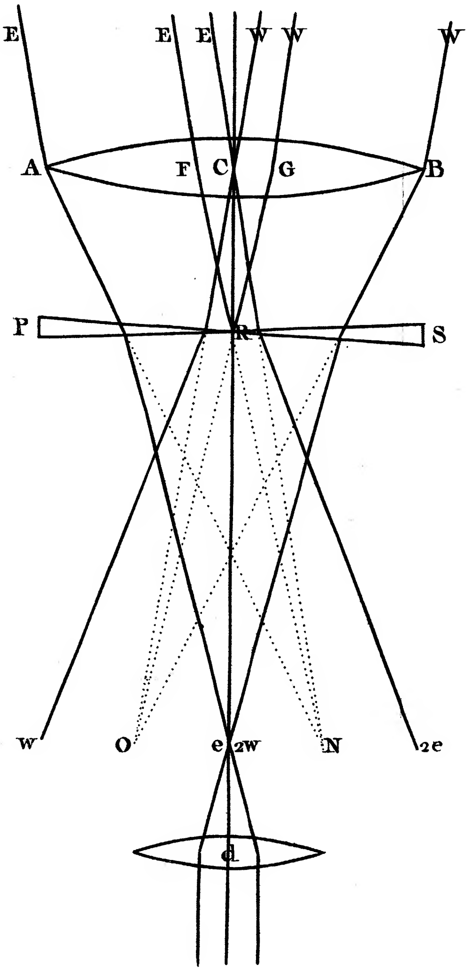


Fig: III.

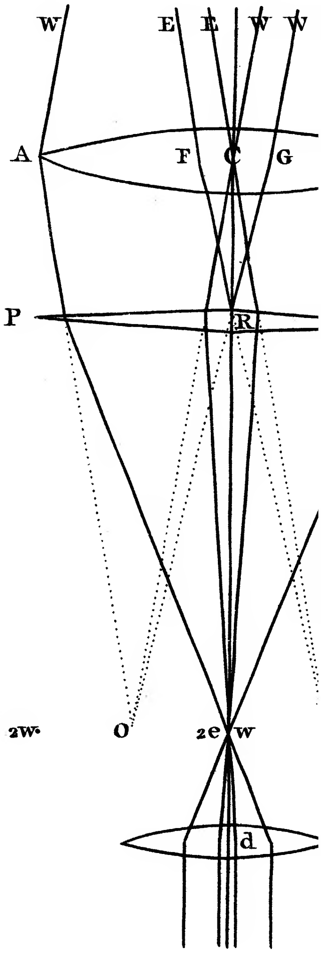
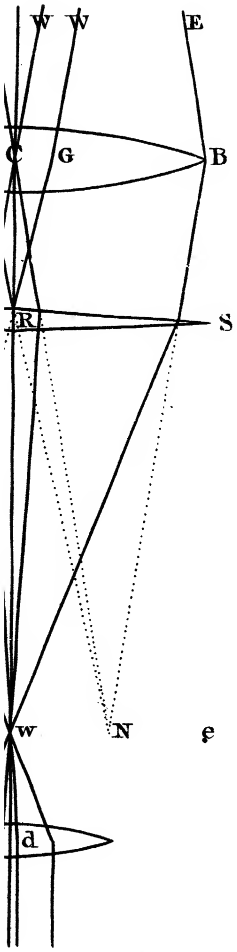


Fig: IV.



IV.

Basire sc.

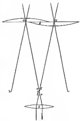


Fig. I

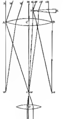


Fig. II

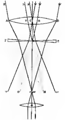


Fig. III

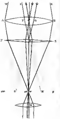


Fig. IV